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(8,0) Quantum mechanics and symmetry enhancement in type I' superstrings

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Abstract: The low-energy supersymmetric quantum mechanics describing D-particles in the background of D8-branes and orientifold planes is analyzed in detail, including a careful discussion of Gauss' law and normal ordering of operators. This elucidates the mechanism that binds D-particles to an orientifold plane, in accordance with the predictions of heterotic/type I duality. The ocurrence of enhanced symmetries associated with massless bound states of a D-particle with one orientifold plane is illustrated by the enhancement of $SO(14)\times U(1)$ to E_8 and $SO(12)\times U(1)$ to E_7 at strong type I' coupling. Enhancement to higher-rank groups involves both orientifold planes. For example, the enhanced $E_8\times E_8\times SU(2)$ symmetry at the self-dual radius of the heterotic string is seen as the result of two D8-branes coinciding midway between the orientifold planes, while the enhanced $SU(18)$ symmetry results from the coincidence of all sixteen D8-branes and $SO(34)$ when they also coincide with an orientifold plane. As a separate by-product, the s-rule of brane-engineered gauge theories is derived by relating it through a chain of dualities to the Pauli exclusion principle.

Keywords: Superstrings and Heterotic Strings, String Duality, Superstring Vacua, Branes in String Theory.

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Contents

1 Introduction

The dynamics of D-particles in the presence of D8-branes and orientifold planes plays an important part in type I' superstring theory [1], which is the compactification of M-theory on a cylinder [2]. Several features of the associated (8,0) supersymmetric quantum mechanics[1] were introduced in [3, 4, 5, 6] and further discussed in the context of heterotic/type-I duality and of the matrix conjecture in [7, 8, 9, 10, 11, 12, 13, 14]. In this paper several aspects of the dynamics of this system will be considered in more detail. Particular care will be taken with issues relating to normal ordering, the Gauss law constraints on physical states and the effect of the anomalous creation of fundamental strings [15, 16, 17, 18, 19, 20] when a D-particle crosses a D8-brane. Several of our conclusions differ from those in the previous papers on this subject. We will be lead to a rather subtle pattern of level crossing as the moduli of the system are varied. This will elucidate the mechanism responsible for binding D-particles to orientifold planes, as required by heterotic-type I duality. The expected enhanced symmetries of the type I' theory [1] at strong coupling arise explicitly at points where certain bound-state masses vanish.

[1] The notation $(8,0)$ defines the number of left-moving and right-moving supersymmetries in the T-dual $(1+1)$-dimensional field theory describing strings in the presence of D9-branes.

In section 2 we will review the quantum mechanical formulation of the dynamics of D-particles in the type IIA superstring theory in the presence of a D8-brane. The quantum-mechanical approximation to the full dynamics should be good for slowly-moving D-particles and is motivated by explicit string calculations. A D8-brane acts like a domain wall and separates regions of space in which the mass parameter, m, of the IIA theory differs by one unit. We will see that the value of m is correlated, via Gauss' law, with the number of strings joining the D-particle to the D8-branes, which is in accord with the picture that a string is created or destroyed when a D-particle passes through a D8-brane. The fact that these strings have a unique BPS ground state which is fermionic leads, via a chain of dualities, to an understanding of the s-rule of Hanany and Witten [21, 22, 23, 24]. This rule states that a configuration with more than one D3-brane joining a NS-fivebrane and a D5-brane, and extending in all space-time dimensions, cannot have a quantum state with unbroken supersymmetry.

Our considerations are extended in section 3 to type I' D-particle quantum mechanics, which is a projection of the type IIA theory. This is again motivated by string calculations, now in nine dimensions and in the presence of two orientifold planes and sixteen parallel D8-branes with their mirror images. The presence of states of non-zero winding restrict the region of moduli space in which it is consistent to use the quantum mechanical truncation of this system. This is the region in which one of the orientifold planes is at the origin in the transverse direction with half the D8-branes close by while the other orientifold and the remaining D8-branes are taken to infinity. A single D-particle in this background is necessarily stuck to the orientifold plane and generically has sub-threshold BPS bound states with masses that are easy to determine. We will also discuss the states of a probe D-particle and its mirror as they move in the transverse directions. Gauss' law again leads to strong constraints on the pattern of strings joining the D-particle to the various branes and again there are explicit sub-threshold BPS bound states of the D-particle pair with the orientifold plane with easily calculable masses.

In section 4 we will see that the masses of these type I' BPS sub-threshold bound states have precisely the values expected on the grounds of heterotic/type I duality. We will first discuss symmetry enhancement involving only one orientifold and its nearby eight D8-branes, which can be studied consistently within the quantum mechanical approximation to the system. We will find agreement with the predictions of duality. The description of symmetry enhamcement in the region of moduli space in which the two orientifold planes are at finite separations typically requires the inclusion of string winding modes and falls outside the range of validity of the simple quantum mechanical approximation to type I' dynamics. Nevertheless, it is straightforward to determine the physical ground states in this region by continuing from the region in which quantum mechanics is valid for each orientifold independently. An example of such an enhancement is the point at which the symmetry becomes $E_8 \times E_8 \times SU(2) \times U(1)$. This is the symmetry of the nine-dimensional heterotic string compactified to its self-dual radius in the absence of any Wilson lines. We will find this symmetry arising in the type I' picture when seven D8-branes (and their mirrors) coincide with each orientifold plane and the remaining two D8-branes coincide at a point in between the orientifold planes (with similar coincidence of their mirrors). The symmetry enhancement to $SU(18) \times U(1)$ can be likewise seen to arise when all sixteen D8-branes coincide half-way between the two orientifold planes. Another special example is the point at which all D8-branes coincide with one orientifold plane and the symmetry is enhanced to $SO(34) \times U(1)$ at a particular value of the coupling.

2 D-particles and D8-branes

In this section we will reconsider the dynamics of D-particles interacting with a D8-brane. Although many of our considerations only depend on the low-energy quantum-mechanical degrees of freedom arising from the string ground-state degrees of freedom, it is essential for some purposes to bear in mind that the complete string theory also contains winding states and oscillator excitations.

2.1 (8,8) Quantum mechanics

Consider first a cloud of n isolated type-IIA D-particles moving slowly in the ten-dimensional IIA vacuum. They are described at sub-stringy energies by the usual supersymmetric $U(n)$ quantum mechanics [25, 26, 27] with lagrangian

$$\mathcal{L} = \frac{1}{2}\,\mathrm{tr}\,\Big(DY_j DY_j + \frac{g^2}{2}[Y_i, Y_j]^2 + i\; S_A DS_A + g\; S_A(\Gamma^0\Gamma^j)_{AB}[Y_j, S_B]\Big)\;, \tag{2.1}$$

which is invariant under the (8,8) supersymmetry transformations,

$$\begin{aligned}
\delta Y^j &= i\xi_A(\Gamma^0\Gamma^j)_{AB}S_B\\
\delta A_0 &= -i\xi_A S_A\\
\delta S_A &= DY_j(\Gamma^0\Gamma^j)_{AB}\xi_B + \frac{ig}{4}[Y_i, Y_j][\Gamma^i, \Gamma^j]_{AB}\xi_B .
\end{aligned} \tag{2.2}$$

The covariant derivative of any of the hermitian $n \times n$ matrices, Y_j, S_A and A_0, is defined by $DM = \dot{M} + ig[A_0, M]$. Under the $SO(1,9)$ Lorentz group $i, j = 1,..,9$ are spatial-vector indices, and $A, B = 1,..,16$ are Weyl-Majorana spinor indices. The ten-dimensional Γ-matrices are purely imaginary and satisfy the algebra $\{\Gamma^\mu, \Gamma^\nu\} = -2\eta^{\mu\nu}$ with metric signature $(-+...+)$. The matrix coordinates appearing in the lagrangian and the detailed form of the above expressions originate from the supersymmetric Yang–Mills theory that describes the ground states of the (00) strings joining pairs of D-particles.

Canonical quantization in the $A_0 = 0$ gauge leads to the commutation relations

$$\begin{aligned}
[\Pi_i^{rs}, Y_j^{tq}] &= -i\;\delta_{ij}\delta^{rq}\delta^{st}\\
\{S_A^{rs}, S_B^{tq}\} &= \delta_{AB}\delta^{rq}\delta^{st}\;,
\end{aligned} \tag{2.3}$$

where $\Pi_j = \dot{Y}_j$. The supercharges and the hamiltonian read

$$\begin{aligned}
Q_A &= (\Gamma^0\Gamma^j)_{AB}\;\mathrm{tr}(\Pi_j S_B) + \frac{ig}{4}[\Gamma^i, \Gamma^j]_{AB}\;\mathrm{tr}([Y_i, Y_j]S_B)\\
\mathcal{H} &= \frac{1}{2}\;\mathrm{tr}\;\Big(\Pi_j^2 - \frac{g^2}{2}[Y_i, Y_j]^2 - g\; S_A(\Gamma^0\Gamma^j)_{AB}[Y_j, S_B]\Big),
\end{aligned} \tag{2.4}$$

while Gauss' law is the matrix equation

$$\mathcal{G} = -\frac{\delta \mathcal{L}}{\delta(gA_0)} = i\,[\Pi_j, Y_j] - S_A S_A - n\,\mathbf{1} = 0\ . \tag{2.5}$$

The supersymmetry algebra that follows from (2.4) is given by

$$\{Q_A, Q_B\} = 2\delta_{AB}\mathcal{H} - 2g(\Gamma^0\Gamma^i)_{AB}\,\mathrm{tr}(Y_i\mathcal{G})\ , \tag{2.6}$$

and reduces to standard form in the subspace of physical states, $<\mathrm{phys}'|\mathcal{G}|\mathrm{phys}>= 0$.

An important fact about these expressions is that they are free from operator-ordering ambiguities. This is manifest for the supercharges Q_A. The potential ambiguity in the hamiltonian would have to be linear in Y^i and is therefore forbidden by Lorentz invariance. Closure of the algebra then uniquely fixes the ordering in Gauss' law to be the one of equation (2.5). The piece proportional to the identity in this expression arises because there are nine bosonic and sixteen fermionic terms that need to be rearranged in $\delta\mathcal{L}/\delta A_0$. The addition of a Chern–Simons term, $\int \mathrm{tr}A_0$, is not allowed by the supersymmetries of the problem. In any case its presence would lead to another inconsistency. The canonical commutation relations imply the operator identity

$$\mathrm{tr}\,\mathcal{G} = 0\ , \tag{2.7}$$

which is consistent with the fact that there are no states charged under the abelian $U(1)$ factor of $U(n)$. The addition of a Chern–Simons term in the action would lead to an additional term proportional to the identity in this equation which would then have no solutions. As a result there are no ambiguous operator-ordering parameters in this system of isolated D-particles.

2.2 Adding a D8-brane

Let us now introduce a D8-brane transverse to the ninth direction. This breaks the rotational symmetry from $SO(9)$ to $SO(8)$. Henceforth we will reserve the indices i, j for $SO(8)$ vectors only. A ten-dimensional Weyl-Majorana spinor decomposes as $8_s \oplus 8_c$. In this basis

$$\Gamma^1 \ldots \Gamma^8 = \pm\Gamma^0\Gamma^9 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\ ,$$

where the ten-dimensional chirality is + for S_A and ξ_A, and − for Q_A (in order that $\bar{\xi}Q$ be non-zero). More explicitly, the matrix supercoordinates of the cloud of D-particles are

$$(Y_i, Y_9) \quad \text{and} \quad (S_a, S_{\dot{a}})\ ,$$

where S_a ($S_{\dot{a}}$) has positive (negative) chirality under the broken $SO(1,1)$ that mixes the ninth coordinate and time.[2] The supersymmetry algebra (2.6) can be similarly decom-

[2]The $SO(1,1)$ symmetry can be formally restored by compactifying the ninth coordinate on a circle of vanishing radius. Under a T-duality this configuration is equivalent to a system of n parallel, infinite D-strings inside a nine-brane. This background is however inconsistent, because the nine-brane flux has nowhere to escape.

posed as

$$
\begin{aligned}
\{Q_a, Q_b\} &= 2\delta_{ab}\ (\ \mathcal{H} +\ g\ \mathrm{tr}(Y_9\mathcal{G})\ ) \\
\{Q_{\dot a}, Q_{\dot b}\} &= 2\delta_{\dot a\dot b}\ (\ \mathcal{H} -\ g\ \mathrm{tr}(Y_9\mathcal{G})\ ) \\
\{Q_a, Q_{\dot b}\} &= 2g\ \gamma^i_{a\dot b}\ \mathrm{tr}(Y_i\mathcal{G})\ ,
\end{aligned}
\tag{2.8}
$$

where the $\gamma^i_{a\dot b}$ are the real matrices defined in [28].

Consider next the (08) fundamental strings which stretch between the D8-brane and the D-particles. Such strings have eight Neumann-Dirichlet coordinates x^i, one DD coordinate x^9, and one NN coordinate x^0. The super Virasoro constraints can be used to eliminate the oscillator excitations of x^0, x^9 and of their fermionic world-sheet partners. In the Ramond sector the states are built from half-integer moded oscillators $x^i_{n+\frac{1}{2}}$ and $\psi^i_{n+\frac{1}{2}}$, acting on a vacuum which is a spinor of $SO(1,1)$, but a singlet of the $SO(8)$ rotation group. When the D-particle coincides with the D8-brane, the mass of this Ramond ground state is zero. The GSO projection fixes the product of the $SO(1,1)$ chirality times the world-sheet fermion parity. We will choose a convention in which this sign is negative. In the Neveu-Schwarz sector physical states are built with oscillators $x^i_{n+\frac{1}{2}}$ and ψ^i_n $(n > 0)$ acting on a ground state of $(\mathrm{Mass})^2 = 1/2\alpha'$. The Neveu-Schwarz ground state is a spinor of $SO(8)$ but a scalar of $SO(1,1)$. The GSO projection now fixes the product of the world-sheet fermion parity and of the chirality of the $SO(8)$ spinor. The first few low-lying states are listed in table 1.

$(\mathrm{Mass})^2\alpha'$	$SO(8)$ rep.	boson	+ fermion	− fermion
0	$\mathbf{1}$			$\vert\emptyset>_R$
1/2	$\mathbf{8}$	$\vert\emptyset>_{NS}$	$\psi^i_{\frac{1}{2}}\vert\emptyset>_R$	$x^i_{\frac{1}{2}}\vert\emptyset>_R$
1	$\mathbf{8}\otimes\mathbf{8}$	$x^i_{\frac{1}{2}}\vert\emptyset>_{NS}$	$\psi^i_{\frac{1}{2}}x^j_{\frac{1}{2}}\vert\emptyset>_R$	$x^i_{\frac{1}{2}}x^j_{\frac{1}{2}}\vert\emptyset>_R$; $\psi^i_{\frac{1}{2}}\psi^j_{\frac{1}{2}}\vert\emptyset>_R$

Table 1: Spectrum of (08) strings. The mass does not include the contribution from the length of the stretched strings. The states are classified as bosons, + fermions and − fermions, according to their transformation properties under $SO(1,1)$. The GSO projection forces the product of world-sheet fermion parity times the $SO(1,1)$ and $SO(8)$ spinor chiralities to be negative. We have not specified in this table which of the three possible eight-dimensional representations of $SO(8)$ correspond to each state, since supersymmetry transformations do not commute with triality. The strings are oriented so all states are complex.

When the D-particles are close to the D8-brane the light (08) strings are described by a negative chirality fermion (χ_r) in the fundamental representation of $U(n)$. The new effective lagrangian,

$$\mathcal{L}' = \mathcal{L} \ + \ i\chi^\dagger D\chi - g\chi^\dagger Y^9 \chi \ - gm \ \mathrm{tr}(A_0 + Y^9) \ , \tag{2.9}$$

is invariant under half of the sixteen supersymmetries (2.2), namely those corresponding to positive-chirality parameters ξ_a. The fermions χ do not transform. This is characteristic of chiral supersymmetry in two dimensions. The hamiltonian, supercharges and Gauss constraint read

$$\begin{aligned}
\mathcal{H}' &= \mathcal{H} + g\chi^\dagger Y^9 \chi \ + gm \ \mathrm{tr} Y^9 && (2.10)\\
Q'_{\dot{a}} &= Q_{\dot{a}} && (2.11)\\
\mathcal{G}'_{rs} &= \mathcal{G}_{rs} + \chi_s^\dagger \chi_r + m \ \delta_{rs} \ . && (2.12)
\end{aligned}$$

They obey the supersymmetry algebra

$$\{Q'_{\dot{a}}, Q'_{\dot{b}}\} = 2\delta_{\dot{a}\dot{b}} \ (\ \mathcal{H}' - \ g \ \mathrm{tr}(Y_9 \mathcal{G}') \ ). \tag{2.13}$$

Although the χ fermions do not enter in the expressions for the supercharges, the algebra closes because their hamiltonian is cancelled by the Gauss-constraint term on the right-hand-side [4]. All other states in table 1 are in long multiplets of the supersymmetry algebra. The first excited level, for example, can be easily seen to form an irreducible representation with the same content as a (8,8) vector multiplet. It is therefore a long (8,0) multiplet. The only BPS states of the (08) string are those corresponding to the fermion χ. For most of the considerations in this paper we will be concerned with the low-energy limit in which it will be sufficient to consider the effective quantum mechanics of these ground states.

Although we are not presenting the details here, it is very easy to motivate the quantum mechanical hamiltonian, (2.10), from an exact string calculation in which the D-particle potential is determined by a cylindrical world-sheet with one boundary on the D-particle world-volume and the other on the D8-brane. This may be viewed as the partition function for open strings with one end on each brane. As with other BPS systems, this is equivalent to a one-loop open-string calculation in which only the BPS ground-state fermions, χ_r, contribute so the expression coincides precisely with the energy in the quantum mechanical problem described above. The standard stringy calculation of the phase shift [29] shows on the other hand that the v^2 force obtains contributions from excited states [3, 17], consistent with the fact that supersymmetry alone does not determine the metric in the Y^9 direction.

A further important observation is that if the D-particles are changed to anti-D-particles in the above discussion the sign of the GSO projection changes and hence the sign of the chirality of the fermions χ also changes. However, the configuration is still supersymmetric under the (0,8) supersymmetries corresponding to negative-chirality parameters $\xi_{\dot{a}}$. This should be contrasted with other supersymmetric combinations of

Dp-branes and Dp'-branes, where changing the Dp-branes to anti Dp-branes breaks all supersymmetries [30]. Of course, if D-particles and anti-D-particles are simultaneously present in the D8-brane background all sixteen supersymmetries are broken.

2.3 Gauss' law, string creation and the s-rule

One novel feature of the above system is the appearance of the arbitrary parameter m and a corresponding Chern–Simons term which is compatible with (8,0) supersymmetry. Closure of the algebra requires the same ordering for the χ's in the hamiltonian and in Gauss' law. Since a common reordering can be absorbed by shifting the value of m there is no independent operator-ordering ambiguity in this system.[3] Furthermore, when two sets of D-particles are separated in a direction parallel to the D8-brane, cluster decomposition implies that m is independent of the number of D-particles, n. We will identify m with the mass parameter of the type-IIA supergravity [31] in the region to the right of the D8-brane [32, 33].

Let us now focus on a single D-particle in the Born-Oppenheimer approximation where its position is slowly varying. The canonical anti-commutation relation for the 'fast mode',

$$\{\chi, \chi^\dagger\} = 1 \quad , \tag{2.14}$$

can be realized in a two-state space — the 'vacuum' defined by $\chi|\ 0 >= 0$, and the 'one-string' state $\chi^\dagger|\ 0 >= |\ 1 >$. The difference in energy, $E_1 - E_0 = gY_9$, is positive when the particle is on the right of the D8-brane and can be interpreted as the energy of an (oriented) string stretching leftwards from the particle to the brane. However, as the particle crosses to the other side of the D8-brane the roles of the two states are exchanged: $|\ 1 >$ becomes the new ground state of the system, while $|\ 0 >$ should be interpreted as a state with an oriented string stretching leftwards from the D8-brane to the particle. Therefore, a fundamental string is created or destroyed when a D-particle is moved adiabatically through a D8-brane. What makes this phenomenon inevitable is the fact that both states cannot be physical at the same time. Gauss' constraint,

$$(\chi^\dagger\chi + m)\ |\ \mathrm{phys} >= 0\ , \tag{2.15}$$

is satisfied by the state $|\ 0 >$ if $m = 0$, and by the state $|\ 1 >$ if $m = -1$. In either case the physical state is unique and cannot therefore change during the process.

In either case, furthermore, the physical D-particle feels no (velocity-independent) force as it moves slowly through the D8-brane. This follows rather trivially from the expression for the lagrangian, (2.9). Because of (8,0) supersymmetry, the position Y^9 only occurs in the combination $(A_0 + Y^9)$ so that Gauss' law $\partial\mathcal{L}'/\partial A_0 = 0$ implies that the force on the D-particle also vanishes. Physically, we may interpret this result as coming from the cancellation of two competing effects, both linear in the displacement Y^9: as the D-particle moves leftwards through the D8-brane the mass of the type IIA supergravity

[3] Supersymmetry also allows an arbitrary Y^9-dependent kinetic term in the direction transverse to the brane [3, 4]. This will not affect our discussion here.

jumps by one unit, and there is a discontinuous change in the slope of the effective inverse string coupling constant. The D-particle, whose effective mass is proportional to the inverse coupling, therefore feels a force that apparently jumps discontinuously. However, this discontinuity in the force is balanced precisely by the tension of the fundamental string which is created or destroyed in the process [16, 17].

For $m \neq 0, -1$ neither of the two states of χ is physical. This apparent inconsistency is an artifact of the quantum-mechanical truncation of the system. The full string theory contains excited (08) strings, which can carry any (positive or negative) integer multiple of the elementary $U(1)$ charge, so that any integer value of m is allowed. Gauss' law fixes the *net* number of strings oriented *towards* the D-particle to equal m (or $m+1$) in the region to the right (left) of the D8-brane. The non-chiral excited strings can flip their orientation as the D-particle traverses the D8-brane, while the chiral χ-string must be either created or destroyed. The origin of the arbitrary parameter m can be traced in string theory to the presence of extra D8-branes located at $Y^9 \to \pm\infty$. Long strings joining these distant D8-branes to the particle can break in the proximity of another D8-brane, as we will see in some explicit examples in the sequel.

The general phenomenon of brane creation, which is an important ingredient of the gauge-engineering constructions of field theory [21, 22], was viewed in [15] as the T-dual of the anomaly inflow argument [34] applied to the intersection domain of a pair of branes [35]. A similar reasoning clarifies the other 'mysterious' rule of gauge engineering, the empirical 's-rule' of Hanany and Witten [21]. The rule states that a configuration with more than one D3-brane joining a NS-fivebrane and a D5-brane, and extending in all space-time dimensions (an s-configuration), cannot have a quantum state of unbroken supersymmetry. This rule resolves an apparent contradiction of string theory with the field-theoretic fact that $(2+1)$-dimensional N=8 U(k) gauge theory with $k > 1$ has no supersymmetric vacuum when one turns on a Fayet-Iliopoulos coupling. We will now relate this s-rule to the Pauli-exclusion principle for the fermionic χ-strings of the previous section. A very different argument for this rule has been presented in ref. [23].

The chain of dualities transforming the system of a D8-brane and a D-particle to the Hanany-Witten configuration [16, 17] starts with three T-dualities that map the D8-brane to a D5-brane and the D-particle to a D3-brane. A further S-duality maps the D5-brane to a NS five-brane, while leaving the D3-brane invariant. Finally two extra T-dualities map this to a configuration containing a NS five-brane and a D5-brane. The fundamental (08) strings transform under this sequence of dualities to D3-branes suspended between the two five-branes. The fact that only a single (08) string can be in its supersymmetric fermionic ground state translates directly to the s-rule of Hanany and Witten. Notice that when the transverse direction is compact the rule does not forbid a BPS state with multiple suspended branes of different winding numbers.[4] Furthermore, there is no restriction on the number of (zero winding) suspended branes if all but one of them are in excited non-

[4]In this case the starting D8-brane configuration is inconsistent without the addition of orientifolds but other dual configurations, such as two orthogonal D4-branes, for which flux can escape to infinity are consistent.

BPS states. Such configurations break supersymmetry but may nevertheless be stable since the Pauli principle does not allow the excited branes to decay into their ground state. At face value this accounts for the absence of a supersymmetric field-theoretic vacuum state in the $U(k)$ theories of [21, 22, 24] when $k > 1$, although the decompactification limit of the T-dualized dimensions may be very subtle.

3 D-particles in type I' string theory

As shown in [1] the orientifold projection on nine-dimensional type IIA superstring theory that leads to the type I' theory introduces two orientifold 8-planes. In addition, the weakly-coupled type I' vacuum state contains sixteen parallel D8-branes together with their sixteen mirror images in the orientifold planes. We will here generalize the earlier discussion to take into account the effect of this background on D-particle dynamics.

In order to justify the restriction to the quantum mechanical system we will consider low-energy processes in which the effects of excited and/or winding strings can be neglected. This can be achieved at weak type I' coupling by taking one orientifold plane together with half the D8-branes to infinity in the ninth direction, while leaving the first orientifold with the remaining D8-branes within a sufficiently small (sub-stringy) neighborhood of the origin. However, as is usual in BPS situations, some of our formulae will continue to hold much beyond the range of parameters in which they would have normally been expected to be valid. This will be confirmed explicitly in the following section, where we will compare our results to the heterotic/type I duality predictions.

3.1 Effects of an orientifold plane

One effect of an orientifold is to project onto the symmetric parts of the supercoordinates Y_j and S_a, and the antisymmetric parts of Y^9, $S_{\dot a}$ and A_0. To avoid confusion we will denote these new matrix-coordinates by

$$X_j^{rs} = Y_j^{\{rs\}} \ , \qquad \Theta_a^{rs} = S_a^{\{rs\}} \tag{3.1}$$

and

$$\Phi^{rs} = i(Y^9)^{[rs]} \ , \qquad V_0^{rs} = iA_0^{[rs]} \ , \qquad \lambda_{\dot a}^{rs} = iS_{\dot a}^{[rs]} \ , \tag{3.2}$$

where { } and [ ] denote symmetrization or antisymmetrization. The world-line gauge symmetry is thus truncated from $U(n)$ to $SO(n)$. The antisymmetric-matrix coordinate Φ describes the displacements of *mirror pairs* of D-particles in the direction transverse to the orientifold plane. Finally the chiral fermions χ_I^r, describing strings that stretch between the D-particles and the D8-branes, transform in the real $(n, 2N)$ representation of $SO(n) \times SO(2N)$, where $2N$ is the number of D8-branes (which will later be taken to be sixteen).

The full string calculation of the potential felt by D-particles in this background now includes non-orientable as well as orientable world-sheets. This means that, in addition to the (88) and (08) cylinder diagrams that entered the IIA theory, Möbius strips must also be included. These describe the effect of the mirror images of the D-particles and implement the (anti)symmetrization described above. As before, the excited string states are non-BPS degrees of freedom and decouple from the low-momentum processes that concern us here. A potentially important effect arises from the fact that strings can wind around the compact ninth dimension giving rise to a rich spectrum of BPS states. Even though these winding states decouple in the limit of large separation of the orientifold planes and will be largely irrelevant in the following, it proves very instructive to include them in the evaluation of the complete string diagrams. The effect of the winding numbers on the individual cylinder and Möbius strip diagrams is to introduce a *quadratic* dependence on the positions of the D-particles, Y^9 (or Φ) into the potential. Such a dependence is inconsistent with $(8,0)$ supersymmetry and, not surprisingly, cancels when all diagrams are added together. This cancellation provides a powerful check on the consistency of the calculation.

The quantum mechanical lagrangian for this system can be expressed as

$$\mathcal{L} = \frac{1}{2}\,\mathrm{tr}\,\Big(DX_jDX_j + \frac{g^2}{2}[X_i,X_j]^2 - D\Phi D\Phi - g^2[\Phi,X_i]^2\Big) + \frac{i}{2}\,\mathrm{tr}\,\Big(\Theta_a D\Theta_a - g\Theta_a[\Phi,\Theta_a] \\ - \lambda_{\dot a}D\lambda_{\dot a} - g\lambda_{\dot a}[\Phi,\lambda_{\dot a}] + 2gX_i\gamma^i_{a\dot a}\{\Theta_a,\lambda_{\dot a}\}\Big) + \; i\,\Big({\chi_I}^TD\chi_I + g\,{\chi_I}^T\Phi\chi_I + gm^{IJ}\,{\chi_I}^T\chi_J\Big)\,. \tag{3.3}$$

This is the truncation of the previous lagrangian, eqs. (2.1) and (2.9), with two notable differences. First there is an extra term proportional to the antisymmetric matrix m^{IJ}, whose skew-symmetric eigenvalues are the positions of the D8-branes, relative to the plane of the orientifold. Secondly, there is no Chern–Simons term since the orthogonal gauge groups generally have no simple $U(1)$ factors. Such a term may seem possible in the special case $n=2$, but it is actually forbidden by the requirement of cluster decomposition for a system of more than two D-particles.

The hamiltonian, Gauss constraint and supersymmetry algebra can be derived similarly with the result,

$$\mathcal{H} = \frac{1}{2}\,\mathrm{tr}\Big(\Pi_j^2 - \Pi_\phi^2 + g^2[\Phi,X_j]^2 - \frac{g^2}{2}[X_i,X_j]^2\Big) + \frac{ig}{2}\,\mathrm{tr}\,\Big(\lambda_{\dot a}[\Phi,\lambda_{\dot a}] + \Theta_a[\Phi,\Theta_a] \\ -2X_j\gamma^j_{a\dot a}\{\Theta_a,\lambda_{\dot a}\}\Big) - ig\,\Big({\chi_I}^T\Phi\chi_I + \; m^{IJ}\,{\chi_I}^T\chi_J\Big), \tag{3.4}$$

$$\mathcal{G} = [\Pi_j,X_j] - [\Pi_\phi,\Phi] + i\Theta_a\Theta_a - i\lambda_{\dot a}\lambda_{\dot a} + i\chi_I{\chi_I}^T\Big|_{antisym} = 0\,, \tag{3.5}$$

and

$$\{Q_{\dot a},Q_{\dot b}\} = 2\delta_{\dot a\dot b}\,\Big(\mathcal{H} - g\,\mathrm{tr}(\Phi\mathcal{G}) + ig\,m^{IJ}\,{\chi_I}^T\chi_J\Big)\,, \tag{3.6}$$

where the Gauss constraint is an antisymmetric-matrix equation. The matrix momenta are given by $\Pi_j = \dot X_j$, and $\Pi_\phi = \dot\Phi$. For completeness we also give the canonical commutation relations, suppressing the obvious $SO(8)$ indices,

$$i[\Pi^{rs},X^{pq}] = \{\Theta^{rs},\Theta^{pq}\} = \frac{1}{2}(\delta^{rp}\delta^{sq} + \delta^{rq}\delta^{sp}) \\ i[\Pi_\phi^{rs},\Phi^{pq}] = \{\lambda^{rs},\lambda^{pq}\} = \frac{1}{2}(\delta^{rp}\delta^{sq} - \delta^{rq}\delta^{sp}) \tag{3.7}$$

$$\{\chi_I^r, \chi_J^s\} = \frac{1}{2}\delta_{IJ}\delta^{rs} \ .$$

An important point is that $SO(n)$ gauge invariance, together with the eight supersymmetries, fix all operator-ordering ambiguities of this system. Both the supercharges $Q_{\dot{a}}$ and the Gauss-constraint operators, which must satisfy the $SO(n)$ Lie algebra, are manifestly free from ordering ambiguities. The supersymmetry algebra (3.6) then uniquely fixes the order of operators in the hamiltonian. The extra term proportional to m^{IJ} in this algebra is central because the χ-fermions commute with the supercharges [4]. Note also that (8,0) supersymmetry again allows an arbitrary kinetic term in the ϕ direction. Under some mild assumptions this will not affect our discussion of slowly-moving, semi-classical D-particles.

The simplest case ($n = 1$) corresponds to a single D-particle that is stuck on the orientifold plane. Since the particle is stuck its transverse kinetic energy must vanish and its total mass should be determined exactly by its potential energy. In order to diagonalize the hamiltonian, we make a unitary change of basis $\chi_I \equiv U_{IJ}\,\chi_J$, such that

$$(UmU^\dagger)_{IJ} \ = \ im_I\delta_{IJ} \ = \ \mathrm{diag}\,(im_1, ...im_N, -im_1, ... - im_N) \ . \qquad (3.8)$$

We shall use the notation $I \equiv (\mathcal{I}, \overline{\mathcal{I}})$, where $\mathcal{I} = 1, .., N$ labels eight independent D8-branes and $\overline{\mathcal{I}}$ labels their mirror reflections. This amounts to the decomposition $2N = N \oplus \overline{N}$ under the $U(N)$ subgroup of $SO(2N)$, with the reality condition $\chi_{\overline{\mathcal{I}}} = \chi_{\mathcal{I}}^\dagger$. The hamiltonian, eq. (3.4), of the D-particle takes the simple non-interacting form

$$\mathcal{H}^{(n=1)} \ = \ \frac{1}{2}\Pi_j^2 + gm_{\mathcal{I}}\,[\chi_{\mathcal{I}}^\dagger, \chi_{\mathcal{I}}] \ , \qquad (3.9)$$

with implicit summation on the index $\mathcal{I}$ and with

$$\{\chi_{\mathcal{I}}^\dagger, \chi_{\mathcal{J}}\} = \frac{1}{2}\delta_{\mathcal{I}\mathcal{J}}. \qquad (3.10)$$

The total mass of the D-particle is then given by

$$M = M_0 + gm_{\mathcal{I}}\,q_{\mathcal{I}}, \qquad (3.11)$$

where M_0 is the 'bare' rest mass of D-particles and $q_{\mathcal{I}} = (\pm\frac{1}{2} ... \pm \frac{1}{2})$ are the weights of the $SO(2N)$ spinor, which realizes the fermionic anitcommuation relations. A discrete Z_2 remnant of the local symmetry, which changes the sign of the χ's, forces this spinor to have definite chirality. If $2\tilde{N}$ D8-branes coincide with the orientifold, the stuck D-particle will describe a degenerate spinor representation of $SO(2\tilde{N})$. By realizing the algebra of the Θ_a zero modes, the particle also carries a vector-supermultiplet representation of the spatial $SO(8)$.

The case $N = 8$ is special because in this case the theory in the bulk is massless type IIA supergravity with freely-propagating gravitons. The bare mass M_0 of D-particles, which are Kaluza-Klein supergravitons, is therefore well defined far from the D8-branes. Since the mass difference $M - M_0$ in (3.11), which depends linearly on the positions of the D8-branes, can be negative, there exist sub-threshold bound states. Consider a specific example that will be of relevance in the next section in which seven D8-branes and their mirrors coincide with an orientifold plane ($m_{\mathcal{I}} = 0$) with a single mirror pair located at $\pm m_1$. The space-time gauge symmetry associated with this configuration is $SO(14) \times U(1)$. From (3.11) it follows that the states divide into two groups with positive and negative binding energies and so they have masses

$$M^{\pm} = M_0 \pm \frac{1}{2} g m_1. \tag{3.12}$$

There are therefore 2^6 degenerate sub-threshold bound states in this configuration which fill out a $(\mathbf{64}, -\frac{1}{2})$ chiral spinor representation of $SO(14) \times U(1)$. Similarly, there is a degenerate state consisting of an anti D-particle stuck on the orientifold plane that describes a $(\mathbf{64'}, \frac{1}{2})$ spinor of the opposite $SO(14)$ chirality. Two D-particles in such a state would not be able to leave the orientifold plane even if they happened to collide.

3.2 Bound states of a mirror pair

In order to further elucidate the mechanism responsible for binding the D-particles to the orientifold, we turn now to the case $n = 2$ which allows for motion of a mirror pair transverse to the orientifold plane. We are interested in the 'Coulomb' branch along which ϕ is non-zero. We will work within a Born-Oppenheimer approximation, and focus attention on the massive 'fast' coordinates, which are those charged under the $SO(2) \simeq U(1)$ gauge symmetry on the world-line. The charges of the various fields are as follows. Antisymmetric matrices are neutral, vectors have charge equal to one, while symmetric matrices contain a neutral trace part and a charge-two complex component. Accordingly, the symmetric matrices may be replaced by the charged creation and annihilation operators,

$$\begin{aligned} \frac{1}{2}(X_j^{22} - X_j^{11}) + iX_j^{12} &= \frac{1}{\sqrt{2}}(a_j + b_j{}^{\dagger}) \\ \frac{1}{2}(\Pi_j^{22} - \Pi_j^{11}) - i\Pi_j^{12} &= \frac{i}{\sqrt{2}}(a_j{}^{\dagger} - b_j) \\ \frac{1}{2}(\Theta_a^{22} - \Theta_a^{11}) + i\Theta_a^{12} &= \theta_a, \end{aligned} \tag{3.13}$$

and the (08) fermions may be complexified,

$$\chi_I^1 + i\chi_I^2 \equiv \chi_I \ . \tag{3.14}$$

The complex fermions, $\chi_{\mathcal{I}}$ and $\chi_{\overline{\mathcal{I}}}$, are now independent and have the same (negative) U(1) charge. The canonical anticommutation relations are

$$[a_i, a_j{}^{\dagger}] = [b_i, b_j{}^{\dagger}] = \delta_{ij} \quad , \quad \{\theta_a, \theta_b^{\dagger}\} = \delta_{ab} \quad , \quad \{\chi_I, \chi_J^{\dagger}\} = \delta_{IJ} \ . \tag{3.15}$$

The string interpretation of these operators is exhibited in figure 1. The operators $a_j^{\dagger}$ and $\theta_a^{\dagger}$ create bosonic and fermionic strings joining the D-particle to its mirror image,

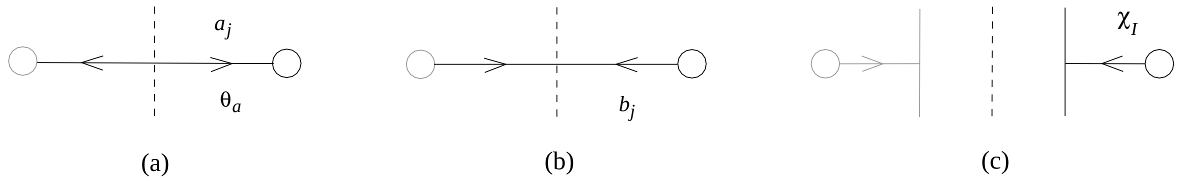


Figure 1: The four types of stretched strings along the Coulomb branch in the n=2 case, as described in the text. Figures (a) and (b) depict the strings stretching between the D-particle and its mirror, while (c) those between the particle and a D8-brane. The broken line is the orientifold. The mirror image of the string in (c) does not contribute to the charge and energy, and is drawn as a faint line. For a given point in the moduli space Gauss' law fixes the net number of arrows pointing towards the particle or its mirror image. The energy of these configurations is proportional to the total (non-faint) string length.

and oriented *towards* the D-particles. The operator $b_j^\dagger$ creates a similar bosonic string, but oriented *away* from the D-particle and its image. Finally, the operator $\chi_I^\dagger$ creates a fermionic string stretching *away* from the D-particle towards the Ith D8-brane if the latter lies between the particle and its mirror image. Otherwise, $\chi_I^\dagger$ annihilates a string stretching from the D8-brane *towards* the D-particle. The orientation of these strings reflects their chirality in the dual type I description, and fixes the sign of their U(1) charge. Gauss' constraint, eq. (3.5), reads

$$\mathcal{G}_{21} \;=\; a_j^\dagger a_j + \theta_a^\dagger \theta_a - b_j^\dagger b_j - \frac{1}{2}\chi_I^\dagger \chi_I - (4 - \frac{N}{2}) \;=\; 0\ . \tag{3.16}$$

The U(1) charges of the various strings, +1, +1, −1 and −1/2, are read off in the order they appear. A mnemonic for Gauss' law is that it fixes the net number of arrows pointing *towards* the D-particle *or* its mirror image. The mirror images of χ-strings do not contribute to this counting, nor to the total energy, and are thus drawn with faint lines in the figurey.

We are now ready to discuss the dynamics of the mirror pair of D-particles. Consider, without loss of generality, the point $\Phi_{21} = \phi > 0$ in moduli space. Neglecting interaction terms, and performing a ϕ-dependent rescaling of the bosonic matrix coordinates, allows us to express the hamiltonian of 'fast' modes as

$$\mathcal{H}^{n=2}_{\rm fast} \simeq 2g\phi\; (a_j^\dagger a_j + \theta_a^\dagger \theta_a + b_j^\dagger b_j) + (\phi - m_I)\chi_I^\dagger \chi_I + g\phi\; (8 - N)\ . \tag{3.17}$$

It is important that the order of operators, and the related subtraction term, is fixed unambiguously in the expressions for $\mathcal{G}$ and $\mathcal{H}$. In the type IIA theory we had the freedom to choose the value of the mass parameter m in the far right asymptotic region, by placing a number of D8-branes at infinity. Since the mass jumped precisely by one unit at each D8-brane, its value everywhere else was fixed. In the present situation there is no ambiguity whatsoever, because the value of the mass is fixed uniquely in the region between the orientifold and the closest (mirror pair of) D8-branes. This is the physical interpretation of the fact that $SO(n)$ quantum mechanics does not allow for a Chern-Simons term.

In the region to the right of all D8-branes the lowest eigenstate of $\mathcal{H}_{fast}$ is the naive 'vacuum'

$$(a_j\ ,b_j\ ,\theta_a\ ,\ \text{and}\ \chi_I)\ |0>=0\ . \tag{3.18}$$

It satisfies Gauss' law if and only if there are precisely $N = 8$ D8-branes (and their mirrors), in which case $\mathcal{H}_{fast}|0>=0$. This is consistent with the well-known fact [1] that the dilaton tadpole cancels between an orientifold and eight D8-branes, so that there is no dilaton gradient in the asymptotic region. Therefore, a distant D-particle has 'no strings attached' to it and feels no static force. If all D8-branes coincide with the orientifold, any bound states of the mirror pair of particles must be threshold bound states. Heterotic/type-I' duality predicts the existence of such bound states [5, 7], but this fact is hard to establish independently.

The problem becomes simpler if at least one pair of D8-branes moves away from the orientifold, in which case our discussion of stuck isolated D-particles suggest the existence of sub-threshold bound states. As a particular explicit example consider again the configuration in which all the D8-branes coincide with the orientifold plane apart from one pair ($m_1 > 0$, $m_{2...8} \simeq 0$). This breaks the $SO(16)$ gauge group associated with (88) open strings to $SO(14) \times U(1)$. It is energetically-favourable for a (pair of) D-particles at position $\phi > m_1$ to be in the state $|0>$ with 'no strings attached' (figure 2a). As it crosses the D8-brane a string is anomalously created (figure 2b). However, in contrast to the earlier discussion of an isolated D8-brane, the quantum mechanics now has a richer spectrum of allowed states. These include the bosonic and fermionic states,

$$|j\ I\rangle =\ a_j^\dagger\ \chi_1^\dagger\chi_I^\dagger\ |0\rangle\ , \quad \text{and} \quad |a\ I\rangle =\ \theta_a^\dagger\chi_1^\dagger\chi_I^\dagger\ |0\rangle\ , \tag{3.19}$$

with $I = 2, ...8$ or $10, ...16$. These are obtained by trading the string attaching the D-particle to the right-most D8-brane for two strings — one that attaches it to its mirror image, and the other to one of the D8-branes on its left (figure 2c).[5] The net number of arrows pointing to the D-particle and to its image is indeed conserved, in agreement with our mnemonic for Gauss' law. Put differently, these states can be created from the 'vacuum' by acting with the $SO(n)$-invariant operators

$$V_{1I}^j \sim\ \text{tr}(X^j\chi_1{\chi_I}^T)\ \ \text{or}\ \ V_{1I}^a \sim\ \text{tr}(\theta^a\chi_1{\chi_I}^T)\ , \tag{3.20}$$

where the χ's are here $SO(n)$ vectors. This guarantees their consistency with the Gauss constraint. The quantum numbers of the states (3.19) are those of a ten-dimensional vector supermultiplet, in the $(\mathbf{14},-1)$ representation of the $SO(14) \times U(1)$ target-space gauge group.

The energy of the fast modes in any of these states, $E_{fast} = g(4\phi - m_1)$, is proportional to the length of stretched strings in figure 2c minus the length of the string in figure 2b, which was annihilated by the action of the above operators on the naive vacuum. A (pair

[5] Positive-chirality strings stretching away from the orientifold transform under the (8,0) supersymmetries, so that at $\phi \neq 0$ these states are not supersymmetric. This is consistent with the fact that there is a net force pushing the D-particles to the orientifold plane.

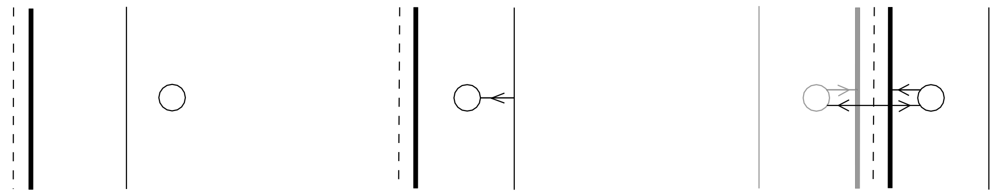


Figure 2: A D-particle pair in the background of an orientifold plane (broken line) and eight D8-brane pairs, as described in the text. Seven D8-brane pairs and their mirror pairs (bold and faint thick lines) sit at the orientifold, while one pair is moved out to position $\pm m_1$. In (a) the particle lies to the right of all D8-branes and has no strings attached. Crossing the outermost D8-brane as in (b) leads to the creation of a string, in accordance with Gauss' law. In (c) it has become energetically favourable to trade this string for two others, one stretching to a D8-brane on the left, and one stretching to the particle's image. The latter carries two units of charge, so the total charge of the state is conserved.

of) D-particles in one of these states will therefore feel a constant force attracting it to the orientifold plane. To leading order at weak string coupling, the mass of the resulting bound state is the total rest mass of the D-particles minus the energy at the bottom of the potential well,

$$M \simeq 2M_0 - gm_1 \ . \tag{3.21}$$

Corrections to this formula may come from the neglected interaction terms, the kinetic energy of the D-particles, and other higher-dimensional operators. However, the fact that the mass in (3.21) is precisely twice the mass of the spinor states in (3.12) suggests that these subleading effects miraculously cancel. This observation is based on comparison with the heterotic string where it is easy to see (as we will in the next section) that the $SO(14) \times U(1)$ vector states are precisely twice as massive as the spinor states.

The transition from the naive vacuum to one of the states (3.19) involves the emission of a massive $SO(16)$ gauge boson, represented by an open string stretching between the D8-brane on the right, and one of the fourteen branes at the orientifold. This is a local interaction that occurs when the D-particle meets the orientifold plane. It is associated with the operators V^j_{1I} and V^a_{1I}, which are the vertex operators for the emission of such massive (super)gauge bosons in the heterotic matrix model. There is however one further subtlety since the quantum-mechanical model also contains the fermionic zero-modes $\lambda_{\dot a}$ and $\mathrm{tr}\Theta_a$, whose Clifford algebra is realized by a 256-dimensional representation of the Lorentz group. It would seem that this representation is carried by the 'vacuum' $|0>$, consistent with the M-theory interpretation of D-particles as massive Kaluza-Klein supergravitons in the bulk of the eleven-dimensional world [2]. Does this mean that the above bound states transform in a 16×256-dimensional representation of the Lorentz group? The answer is no, because the D-particles interact with massless supergravitons, which are the massless closed strings in the bulk. They can carry away even-tensor representations of the transverse $SO(8)$ without changing the semi-classical energy of the system. The same is true for the massless $SO(14)$ gauge bosons, which can be emitted or absorbed freely when the D-particles coincide with the orientifold plane. Since these effects cannot be accounted for within the Born-Oppenheimer treatment of the system the quantum numbers of the bound states can be only fixed modulo emission of massless supergravitons and massless super-gauge bosons.

One important fact should be stressed. What we have established is the existence of states binding the mirror pair of D-particles to the orientifold plane. However, the two particles could still escape to infinity along the orientifold directions $X_j \sim x_j \sigma_3$, i.e. along the Higgs branch of the corresponding moduli space. Comparing the mass formulae (3.12) and (3.21) shows that the vector state is stable against this decay only if there is a threshold bound state of two D-particles stuck on the orientifold. The existence of this threshold bound state will follow from target-space gauge invariance as will be explained in the following section.

3.3 Moving more D8-branes

First however we will consider the more general situation, in which more than one pair of D8-branes is moved off the orientifold plane. To start with let us move two (pairs of) D8-branes to positions m_1 and m_2, leaving all others at the orientifold. The open-string gauge symmetry is broken down to $SO(12) \times U(1) \times U(1)$. A single D-particle stuck at the orientifold plane will carry a $SO(12)$ spinor representation, and have a spectrum of masses

$$M = M_0 \pm \frac{1}{2} g m_1 \pm \frac{1}{2} g m_2 \ . \tag{3.22}$$

The lightest stuck D-particles will therefore be in the representation $(\mathbf{32}, -\mathbf{1/2}, -\mathbf{1/2})$.

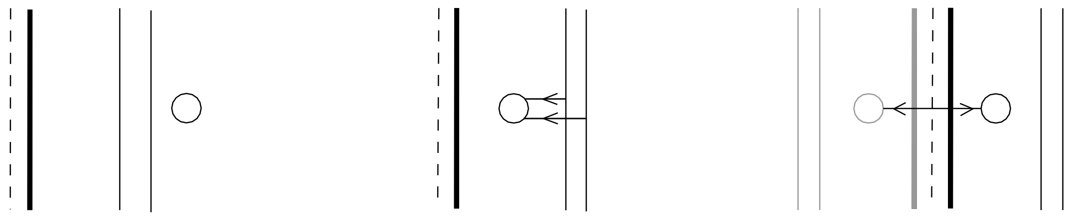
The binding mechanism of a mirror pair in this background is illustrated in figure 3. When the particle lies to the right of the D8-branes it has no strings attached to it as usual (figure 3a). As the D-particle moves to the left, crossing the two D8-branes, two strings oriented towards the D-particle are created (figure 3b). The total energy of the fast modes in this configuration is still zero. The physical states of lowest energy, for sufficiently small ϕ, are now

$$|j\rangle = \ a_j^\dagger \ \chi_1^\dagger \chi_2^\dagger \ |0\rangle \ , \quad \text{and} \quad |a\rangle = \ \theta_a^\dagger \chi_1^\dagger \chi_2^\dagger \ |0\rangle \ . \tag{3.23}$$

They are obtained by trading the two strings attaching the D-particle to the D8-branes on its right for a single string attaching it to the particle's image in the mirror (figure 3c). The energy of the fast modes in this configuration is $E_{fast} = g(4\phi - m_1 - m_2)$, so the rest mass of a bound state to (sub)leading order at weak coupling is

$$M \simeq 2M_0 - gm_1 - gm_2 \ . \tag{3.24}$$

This is again twice the mass of the lowest-mass state of a single D-particle that coincides with the orientifold plane (3.12). Comparison with the heterotic string in the next section will again indicate that the approximate expression for the mass of the two D-particle state is exact.



(a) (b) (c)

Figure 3: A mirror pair of D-particles in the background with two displaced D8-branes. In (a) the particle is to the right of the D8-branes and has no strings attached. In (b) it has crossed to the left, and two strings attaching it to the D8-branes have been created. In (c) these have been traded for a single string attaching the D-particle to its mirror image.

States created by V^j_{1I}, V^j_{2I} $(I \neq 1, 2)$ and their space-time spinor partners are also lower in energy than the naive vacuum $|0>$ near the orientifold plane. They are reached by trading only one of the two strings that attach the particle to a D8-brane on its right. However, such states have higher mass than the states (3.23), and can indeed decay into them by emitting one massive and one massless open-string gauge boson at threshold. All states with $n > 2$ have also similar potential instabilities at threshold. Strictly-speaking, the only conclusions that follow rigorously from our discussion in this section are (i) that the lowest-mass n=1 states are stable, and (ii) that some of the n=2 states cannot decay by emission of a Kaluza-Klein supergraviton.

Situations where more than two D8-branes are displaced from the orientifold plane can be analyzed similarly. One novel subtlety, that will play a role later on, appears when all eight D8-branes are displaced. The n=1 mass spectrum (3.9) depends in this case on whether the number of positive $m_{\mathcal{I}}$'s is even or odd. Put differently, because the D-particle carries a chiral spinor representation of $SO(16)$, only an even number of D8-brane reflections in the mirror leave its spectrum invariant. In particular, there are two inequivalent configurations with eight D8-branes at position m, and their eight mirror images at $-m$. In one configuration there is a unique lowest-lying D-particle state with mass $M = M_0 - 4gm$, in the $(\mathbf{1}, -4)$ representation of the $SU(8) \times U(1)$ open-string gauge group.[6] In the other configuration, the lowest-lying state of the D-particle is in a degenerate $(\mathbf{8}, -3)$ representation, and has mass $M = M_0 - 3gm$. This degeneracy can be pictured as the different ways in which a string can join any of the eight displaced D8-branes to the D-particle (or anti D-particle). We will use this second configuration in the discussion of the special locus of moduli space with $SU(18)$ gauge symmetry.

4 Global aspects of moduli space

A significant test of our understanding of issues concerning bound states in the type I'

[6] Our normalization is such that a string attached to one of the eight D8-branes has U(1) charge equal to ± 1.

theory is whether we can reproduce the spectrum of BPS states in regions of moduli space that are outside of the weakly coupled domain. For example, using the duality relations between type I' string theory and the heterotic string it is easy to see that there must be points in moduli space (at strong type I' coupling) at which the symmetry is enhanced [1]. These correspond to the compactifications of the heterotic string on a circle where the symmetry is enhanced at specific values of the radii and of the background Wilson lines. It was shown in [1] that these enhanced symmetry points occur in the type I' language precisely when the effective couplng constant diverges at one of the orientifold planes. However, this gave no hint of the mechanism that generates the extra massless states needed to enlarge the symmetry. In this section we will see how such enhanced symmetry points can be exactly determined by the D-particle $(8,0)$ quantum mechanical hamiltonian.

4.1 Heterotic string and M-theory on the cylinder

Before describing the type I' dynamics we shall review the expectations based on duality with the heterotic string or M-theory compactified on a cylinder [2] of circumference $2\pi r_{11}$ and length πr_9. This compactification has two dual interpretations (i) as weakly-coupled type I' theory in the limit $r_{11} \to 0$, and (ii) as the weakly-coupled heterotic theory in the limit $r_9 \to 0$. More generally the compactification has a moduli space of dimension eighteen. On the heterotic side the moduli are the coupling constant λ_h, the radius r_{11} of the cylinder, and sixteen Wilson lines in the Cartan subalgebra of the gauge group. On the type I' side they are the coupling constant $\lambda_{I'}$, the separation πr_9 of the orientifolds, and the positions of the sixteen independent D8-branes between the two orientifold planes.

In order to work out the precise mapping between these two parametrizations of moduli space, we first consider the special locus where the gauge symmetry is $SO(16) \times SO(16) \times U(1) \times U(1)$. We have already seen that on the type I' side this is a privileged configuration because it corresponds to sixteen D8-branes sitting precisely at each orientifold. Therefore, the (effective) string coupling is constant over the entire interval and the two length scales of the cylinder can be varied independently without encountering any phase transitions. Since a mirror pair of D-particles is identified with a freely-propagating Kaluza-Klein graviton in the bulk, we have

$$2M_0 = \frac{1}{r_{11}} \ . \tag{4.1}$$

The theory also contains BPS states that are membranes stretching a number w of times between the two orientifolds. If w is odd the corresponding type I' string must start and end at two different D8-branes sitting at opposite ends of the cylinder, and therefore carries charge under both $SO(16)$ factors of the gauge group. If w is even the string may (or may not) close onto itself, and consequently may (but need not) be neutral. The minimum mass of a stretched membrane is $2\pi^2 r_9 r_{11} T_2$, where T_2 is the membrane tension. The ratio of the winding and Kaluza-Klein masses is an important dimensionless parameter

$$c = 2\pi^2 r_9 r_{11}^2 T_2 \ . \tag{4.2}$$

The BPS spectrum of the theory does not depend on the third scale, which can be chosen either as the gravitational coupling, or as the mass of (non-BPS) open-string excitations ($\frac{1}{\sqrt{\alpha_{I'}}} \sim \sqrt{T_2 r_{11}}$).

The two extra $U(1)$ factors of the gauge group are associated with two gauge bosons, $G_{\mu\, 11}$ and $C^{(3)}_{\mu 9\, 11}$, which originate from Kaluza–Klein compactification of the eleven-dimensional metric and the three-form antisymmetric tensor potential. The metric component $G_{\mu 9}$ is projected out of the spectrum by the compactification, and does not give rise to an additional U(1). On the type I' side these fields are the RR one-form coupling to D-particle charge and the Neveu-Schwarz antisymmetric tensor coupling to winding along the ninth direction. In the heterotic theory, they correspond to the off-diagonal components of the metric and of the Neveu-Schwarz antisymmetric tensor, which couple to momentum and winding along the eleventh direction, respectively. Under T-duality momentum gets exchanged with winding and D0-charge with D1-charge.

Let us move on now to the heterotic side, and parametrize first the moduli space in terms of a Wilson line coupling to $E_8 \times E_8$ charge. Such a Wilson line is a sum of two orthogonal eight-vectors, $\mathbf{A} = \mathbf{A_1} \oplus \mathbf{A_2}$, one for each E_8 factor of the gauge group. The states of the theory are defined with reference to the Lorentzian lattice, $\Gamma^{1,17}$, given by

$$(\ p_L \mid p_R\ ) = \left(\frac{\tilde{m}}{r_{11}} - \frac{w r_{11}}{\alpha'_h} \ \Big|\ \frac{\tilde{m}}{r_{11}} + \frac{w r_{11}}{\alpha'_h}\ ;\ \sqrt{\frac{2}{\alpha'_h}}(\mathbf{Q} + w\mathbf{A}) \right) \tag{4.3}$$

where

$$\tilde{m} = m - \mathbf{Q}\cdot\mathbf{A} - \frac{w}{2}\mathbf{A}\cdot\mathbf{A}\ , \tag{4.4}$$

with m and w the integer momentum and winding numbers, $\mathbf{Q} = \mathbf{Q_1} \oplus \mathbf{Q_2}$ a sixteen-component vector in the $E_8 \times E_8$ root lattice $\Gamma^8 \oplus \Gamma^8$, and α'_h the heterotic Regge slope. The E_8 lattice Γ^8 is generated in our conventions by the vectors $\pm e_i \pm e_j$ and $\sum_i \pm\frac{1}{2} e_i$ where e_i form an orthonormal set. For a BPS state there are no oscillator excitations in the left-moving sector, so the mass of the state is

$$M^2 = p_L^2. \tag{4.5}$$

The Wilson line $\mathbf{A}$ takes values in a fundamental cell of the self-dual $E_8 \times E_8$ lattice. In order to make contact with the privileged type I' background, we will choose the center of the fundamental cell to be the point of $SO(16) \times SO(16) \times U(1)^2$ symmetry,

$$\mathbf{A} = \Big(1\ (0)^7\ 1\ (0)^7\ \Big)\ + \mathbf{a}. \tag{4.6}$$

Recall that the root vectors corresponding to the $\mathbf{120}$ of $SO(16)$ have integer entries while those corresponding to the $\mathbf{128}$ have half-integer entries. As a result, at the special point $\mathbf{a} = 0$, the momentum m of states in the $(\mathbf{120}, \mathbf{1})$ and $(\mathbf{1}, \mathbf{120})$ is shifted by an integer, while the momentum of the $(\mathbf{128}, \mathbf{1})$ and $(\mathbf{1}, \mathbf{128})$ is shifted by a half-integer. This breaks the $E_8 \times E_8 \times U(1)^2$ symmetry down to $SO(16) \times SO(16) \times U(1)^2$.

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More generally, substituting (4.6) into (4.3) leads to the following alternative form for the Lorentzian lattice,

$$(p_L \mid p_R) = \left(\frac{n-2\mathbf{q}\cdot\mathbf{a}}{2r_{11}} - w(\frac{r_{11}}{\alpha'_h} + \frac{\mathbf{a}\cdot\mathbf{a}}{2r_{11}}) \,\middle|\, \frac{n-2\mathbf{q}\cdot\mathbf{a}}{2r_{11}} + w(\frac{r_{11}}{\alpha'_h} - \frac{\mathbf{a}\cdot\mathbf{a}}{2r_{11}}) \; ; \; \sqrt{\frac{2}{\alpha'_h}}(\mathbf{q}+w\mathbf{a}) \right) , \tag{4.7}$$

where

$$\mathbf{q} \in \begin{cases} \Gamma^8 \oplus \Gamma^8 & \text{if } w \text{ is even} \\ \\ (1(0)^7\, 1(0)^7) + \Gamma^8 \oplus \Gamma^8 & \text{if } w \text{ is odd}\ , \end{cases} \tag{4.8}$$

and n is even or odd depending on whether we are considering a tensor or a spinor representation of the diagonal $SO(16)$. We can now match precisely heterotic and type I' BPS states for the special value $\mathbf{a} = 0$ by noting that

$$\alpha'_h = (2\pi^2 r_9 T_2)^{-1}\ . \tag{4.9}$$

The heterotic momentum and winding numbers, n and w, must be identified with the number of D-particles and the number of type I' strings stretching between the two orientifold planes, in accordance with the fact that they are charges for the M-theory fields $G_{\mu\, 11}$ and $C^{(3)}_{\mu 9\, 11}$. The vector $\mathbf{q}$ gives the charges under the open-string gauge group on the type I' side. Note, for instance, that type I' states with n odd have at least one D-particle stuck at the orientifold plane and carry a spinor representation of the diagonal $SO(16)$, in agreement with the above heterotic spectrum. Furthermore, type I' states with odd w are strings stretching between two D8-branes at opposite ends of the cylinder and therefore carry one vector index of each $SO(16)$, as well as extra charges in the $E_8 \times E_8$ lattice. This is again in agreement with the heterotic spectrum (4.7). Note finally that the BPS spectrum of the heterotic string does not depend on the string coupling λ_h, which sets the value of the Planck mass. Unlike the tension of type I' strings, the heterotic string tension cannot be varied, however, independently, because $r_{11}^2/\alpha'_h = c$.

4.2 Enhanced symmetry and phase transitions

In order to make contact with the quantum mechanical system of section 3, we would now like to go to the limit in which the orientifold planes are infinitely far apart. This is the limit in which stretched membranes are much heavier than Kaluza-Klein excitations and decouple, which is the case if

$$c \gg 1\ , \quad \text{or equivalently} \quad r_{11} \gg \sqrt{\alpha'_h}\ . \tag{4.10}$$

The states that survive in this limit on the heterotic side have $w = 0$ and $\alpha'_h(p_R^2 - p_L^2) = 0$ or 4. These are precisely the Kaluza-Klein excitations of the supergravity and $E_8 \times E_8$ super Yang-Mills theory, compactified to nine dimensions on a circle. All heterotic string excitations decouple, and so do all states charged under both E_8 factors of the gauge group. We may therefore restrict our discussion to one of these factors, which translates in type I' language to focusing attention on a single orientifold plane.

Strictly speaking the limit (4.10) does not by itself justify the quantum mechanical truncation of the theory. To stay at weak type I' coupling, we should approach this limit by taking $r_9 \to \infty$ while keeping r_{11} small, in units of the eleven-dimensional Planck scale. Furthermore, we must consider processes sufficiently close to the orientifold plane, for which open-string excitations may be neglected. As usual with BPS statements, these caveats will not be necessary, and the mass formulae of the previous section will extend much beyond their naive range of validity. The only important limitation comes from decoupling of the winding BPS states, which are described by a two-dimensional field theory, and this decoupling is guaranteed by the condition (4.10).

The moduli space of the heterotic theory in this limit can be explored by turning on a Wilson line $\mathbf{a} = (a_1 ... a_8)$, for one of the E_8 factors of the gauge group. This Wilson line can be confined to a fundamental cell of the Γ^8 lattice, since two backgrounds that differ by a lattice vector are gauge equivalent. A two-dimensional section of such a cell, centered around the point of $SO(16)$ symmetry, and corresponding to a Wilson line of the form $(a_1\ a_2\ (0)^6\ )$ is shown in figure 4. Using the heterotic mass formula, $M^2 = p_L^2$, with $w = 0$ gives the following spectrum of BPS states,

$$M = \frac{1}{2r_{11}}\,|n - q_{\mathcal{I}} a_{\mathcal{I}}| \tag{4.11}$$

where n is even for the supergravitons and $SO(16)$-adjoint states, and odd for the states in the spinor representation of $SO(16)$. For special values of the Wilson lines, $a_{\mathcal{I}}$, new BPS massless states arise, signalling the standard symmetry enhancements of the heterotic string.

The heterotic mass formula (4.11) agrees precisely with the spectrum of states in the type I' picture if we identify the Wilson lines and D8-brane positions by the relation,

$$g m_{\mathcal{I}} = a_{\mathcal{I}}/2r_{11}\ . \tag{4.12}$$

The exactness of the heterotic expression strongly suggests that our quantum mechanical mass formula (3.9) is also exact. It can therefore be continued to a region in which the binding energy of the D-particle is of the same order as its bare rest mass, in which case the bound state can be massless. For such critical displacements, the type I' gauge symmetry will be non-perturbatively enhanced. This is to be distinguished from the standard perturbative symmetry enhancement in type I' that arises when D8-branes coincide with each other or with an orientifold.

It will be important in the following that all symmetry enhancements actually occur at the boundary of a single cover of moduli space. Such a cover, which is represented by the shaded region in figure 4, is obtained by modding out by the Weyl reflections of the fundamental cell. These Weyl reflections correspond to the permutation symmetries of the D8-branes. For instance the reflection $m_1 \to -m_1$ is simply the exchange of the first D8-brane with its mirror image, together with an odd number of mirror exchanges of the D8-branes sitting at the orientifold. The reflection $m_1 \leftrightarrow m_2$ corresponds to interchanging the first two mirror pairs of D8-branes. At a generic point in the interior of this region

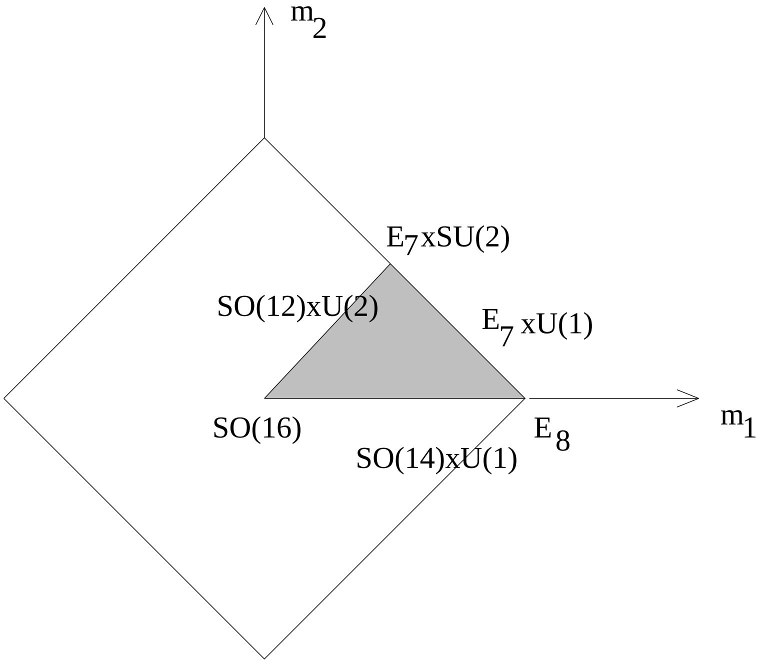

Figure 4: The two-dimensional section of the fundamental cell of moduli space discussed in the text. It corresponds to heterotic Wilson lines of the form $(a_1\ a_2\ (0)^6)$, or, in type I' language, to the motion of two D8-branes away from the orientifold. The center of the cell is the point of $SO(16)$ symmetry where the D8-branes sit at the orientifold. The shaded region is a single cover of the moduli space, obtained by modding out with the Weyl symmetry, i.e. the permutations of the D8-branes. The generic $SO(12) \times U(1)^2$ symmetry in the interior of the cell is enhanced at the fixed loci of the Weyl group, as well as at the outer boundaries. The former enhancement is produced in type I' language by colliding D8-branes, while the latter by massless bound states of D-particles with the orientifold. The type I' description cannot be continued analytically beyond this outer boundary.

the symmetry is $SO(12) \times U(1) \times U(1)$. Along the fixed lines of the Weyl reflections — the two interior boundaries of the shaded region — the symmetry is enhanced in the standard perturbative manner while along the exterior boundary it is enhanced due to the appearance of massless D-particle bound states.

We will now describe some of these points of enhanced symmetry associated with a single orientifold in more detail. For example, we saw in section 3 that when a single mirror pair of D8-branes is displaced from an orientifold there are sub-threshold bound states of the symmetry $SO(14) \times U(1)$ associated with that orientifold. These consisted of the 128 states in a non-chiral $SO(14)$ spinor and 28 states in a complex vector. The masses of these states, given by (3.12) and (3.21), vanish when the displacement of the D8-branes reaches the critical value $m_1 = 2M_0/g$. This is a point of enhanced symmetry where the extra massless states combine with the 92 states of the adjoint of $SO(14) \times U(1)$ to form the **248** of E_8. Moving two mirror pairs of D8-branes gives an enhanced symmetry $E_7 \times U(1)$ at a generic point on the boundary of figure 4. Generally, when N D8-branes and their mirrors are displaced, the unbroken $SO(16-2N) \times U(1)^N$ gauge symmetry in the interior of the cell is enhanced to $E_{9-N} \times U(1)^{N-1}$ at the boundary. Here the groups E_1, E_2, E_3, E_4, and E_5 are conventionally defined as $SU(2)$, $SU(2)\times U(1)$, $SU(3)\times SU(2)$, $SU(5)$ and Spin 10, respectively. One special point on the boundary of the fundamental cell corresponds to the heterotic Wilson line $\mathbf{a} = \left(\left(\frac{1}{6}\right)^7 \; -\frac{1}{6}\right)$ where the symmetry is enhanced to $SU(9)$. In type I' language this arises when all D8-branes are displaced to a critical distance from the orientifold in a such a way that the $SU(8) \times U(1)$ perturbative symmetry is enhanced. The extra massless states are the D-particle bound states in the $(\mathbf{8}, -3)$ representation of the perturbative group discussed at the end of section 3.

The semi-classical type I' description cannot be analytically continued beyond the boundary of the fundamental cell, where some D-particles have binding energy that exceeds their mass. This is in contrast with the heterotic theory (in the limit (4.10)) which can be continued to any point $\mathbf{a}$. However, since the Wilson lines have the periodicity of the E_8 root lattice, any such point can be brought back to the fundamental cell provided one shifts simultaneously the momentum of the various states. In the heterotic description the phase transitions arise when energy levels of fundamental quanta become negative, which is dealt with by a redefinition of the vacuum state. In type I' language this would necessitate second-quantization of the D-particle bound states.

The picture of massless bound states makes qualitative contact with the description of the type I' theory based on classical supergravity [1]. There the coupling varies over space, and diverges at one of the orientifold planes when the symmetry on the heterotic side is enhanced. The supergravity coupling is to be identified with the inverse of the *effective* mass of a D-particle, which varies as the particle moves through the potential. At an enhanced symmetry point the effective mass of a D-particle that is bound to an orientifold plane vanishes so that the effective coupling diverges on that plane. However, the description in terms of D-particle quantum mechanics captures the exact short-distance physics and allows precise calculations of the dynamics at enhanced symmetry points.

4.3 Closing the orientifold gap

Our discussion in the previous subsection was confined to a region of moduli space where the two orientifold planes were very far apart. The physics of symmetry enhancement could thus be discussed by focusing on one orientifold and ignoring the presence of the other. However, some of the interesting phenomena in heterotic string theory involve winding heterotic $E_8 \times E_8$ states, and necessitate the discussion of both orientifolds simultaneously. This is the case, for example, for the special regions in moduli space where the symmetry is enhanced to $E_8 \times E_8 \times SU(2)$, $SU(18)$ or $SO(34)$. These only have one free modulus, the heterotic coupling constant λ_h, and one may wonder whether they can be attained in the type I' picture. Seiberg and Morrison have in fact argued that in order to describe some of these regions one may need to introduce one or two extra D8-branes [37]. We will present some persuasive arguments that such regions can also be described in terms of massless D-particle bound states. In order to give a complete discussion we would have to go beyond the quantum mechanical picture that applies when only one orientifold plane is present to a $(1+1)$-dimensional field theory description that takes into account the infinite tower of type I' open and closed strings winding around the compact ninth dimension. However, we will only present a qualitative overview of the mechanism for symmetry enhancement.

The explicit examples of enhanced symmetries to be considered below will make use of the following geometrical property of the heterotic string. Define the moduli space of lorentzian $\Gamma^{1,17}$ lattices to be $\mathcal{M}$ and the subspace in which the gauge symmetry is larger than $U(1)^{17} \times U(1)_g$ to be $\mathcal{M}_E$, where the factor $U(1)_g$ denotes the abelian gravi-photon symmetry which can never be enhanced. The property we shall use is that $\mathcal{M} - \mathcal{M}_E$ is *connected* (which follows, for example, from [42] and references therein). What makes this statement non-trivial is the fact that some regions of enhanced symmetry have codimension one, and could conceivably separate moduli space into several disconnected components. However, this is not the case because such regions are boundaries, as illustrated in figure 4 for example, where all symmetry enhancements occur at the boundaries of the shaded region.[7] Another simple example is the moduli space of $\Gamma^{2,2}$ lattices, corresponding to compactifications of type II theory on a two-torus. If T and U are the complex structure and Kähler moduli of the torus, then all symmetry enhancements occur on the locus $T = U$. This is again on the boundary of moduli space because of the discrete symmetry $T \leftrightarrow U$.

What this 'theorem' implies in the cases we are interested in is that an arbitrary point in the moduli space of interest can be reached from any other point continuously without ever encountering a phase transition. This guarantees that we may extend the type I' description to cover the Narain moduli space of the perturbative 9d heterotic string and to reach, in particular, the neigbourhood of all the special points mentioned above, without ever encountering negative mass solitonic states. There are additional disconnected regions of the full M-theory moduli space, such as those discussed for instance in [38, 39, 40, 41], which we are not considering in this paper.

For example, consider a type I' configuration with $E_8 \times E_8 \times U(1) \times U(1)_g$ symmetry, obtained by displacing one (pair of) D8-branes to a critical distance $m_{cr} \sim (r_{11}^2 T_2)^{-1}$ from each orientifold plane. There are two free parameters, the radii r_9 and r_{11}, provided the latter is sufficiently small. Decreasing the separation πr_9 of the orientifolds, or equivalently the radius r_{11} of the circle, brings the two separated D8-branes closer. At a critical value,

$$r_9 r_{11}^2 \sim T_2^{-1}, \tag{4.13}$$

the two displaced D8-branes coincide (see figure 5). This gives rise to an extra complex massless gauge boson, corresponding to a stretched type I' string, and enhancing the $U(1)$ factor of the gauge group to $SU(2)$. This corresponds to the straightforward compactification of the heterotic $E_8 \times E_8$ theory, with no Wilson lines, on a circle with self-dual radius.

[7] We thank K. Narain and G. Moore for discussions on this point.

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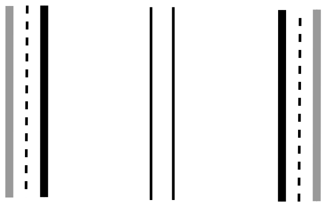

Figure 5: The type I' vacuum close to the point with $E_8 \times E_8 \times SU(2) \times U(1)_g$ symmetry. It is obtained by first displacing the two D8-branes to a critical distance from their respective orientifolds, so as to enhance each $SO(14) \times U(1)$ to an E_8, then closing the orientifold gap so as to make them collide.

A possible source of confusion is the identification of the $U(1)$ factors that complete the three enhanced simple groups, two E_8's and a $SU(2)$. From the heterotic point of view, however, their identification is clear. At a generic point in moduli space the gauge field action has the form

$$\mathcal{L}_{gauge} = -\frac{1}{4} M^{A}_{\ C} \eta^{BC} F_A F_B, \tag{4.14}$$

where F_A $(A = 1, \ldots, 18)$ are the abelian field strengths of the $U(1)$ factors, η^{BC} is the $SO(1,17)$-invariant metric and $M^{A}_{\ B}$ is a moduli-dependent Lorentz boost. The matrix M describes the deformation of the Lorentzian lattice, (4.7), as the Wilson lines are turned on,

$$\begin{pmatrix} p_L \\ p_R \end{pmatrix}_{\mathbf{a}} = M^{-1} \begin{pmatrix} p_L \\ p_R \end{pmatrix}_{\mathbf{a}=0}. \tag{4.15}$$

The perturbative open-string gauge group and the two extra bulk $U(1)$'s (the $C^{(3)}_{\mu 9 11}$ and $G_{\mu\, 11}$ gauge potentials) that couple to Chan-Paton charges, D-particle number and winding, respectively, do not mix at $\mathbf{a} = 0$. However, as the Wilson lines are turned on, or equivalently as the D8-branes are displaced, the U(1)'s generally mix. In particular, the charge vector of the $w = \pm 1$ string stretching between the two displaced D8-branes is rotated in such a way that it becomes orthogonal to the charges of the D-particle bound states at each orientifold plane, in accordance with the fact that these states complete different simple factors of the enhanced gauge group.

In order to describe the $SU(18)$ point we start from the region with $SU(9) \times SU(9) \times U(1) \times U(1)_g$ symmetry and with the orientifolds far apart. As we saw earlier, this requires a configuration in which there is a stack of eight coincident D8-branes displaced a critical distance from each orientifold plane. Reducing the separation of the orientifold planes as in the previous example again results in symmetry enhancement when the two stacks coincide. Among the massless states are the $U(16)$ open-string gauge bosons that arise in the standard manner. Recall that each $SU(9)$ factor contains a massless D-particle bound state in a complex $\mathbf{8}$ of the perturbative $U(8)$ subgroup which can be pictured as the different ways in which a string can join the D-particle (or anti D-particle) to any of the eight displaced D8-branes. Such a string is required to be present by the chirality argument given earlier. When the two stacks of D8-branes coincide the $\mathbf{8}$ is augmented to a **16** since the string can now terminate on any of the coincident branes. Since there can be a D-particle on either orientifold plane there are two complex **16**'s giving a total of 64 extra massless states. In addition, there is a complex $U(16)$ singlet where one D-particle is stuck on each orientifold and they are joined by a χ string. This gives a total of 66 states which completes the 258 states of the adjoint of $U(16) \times U(1) \times U(1)_g$ to $SU(18) \times U(1)_g$.

Finally, we may consider the mechanism for the enhancement of the symmetry to $SO(34) \times U(1)$. The point in moduli space where this occurs can be reached by starting from the $SO(16) \times SO(16)$ configuration and displacing the stack of eight D8-branes (and their mirrors) from the left orientifold plane. When the coupling constant is sufficiently small this set of displaced D8-branes can be moved onto the right orientifold plane without encountering a phase transition. Now all 32 D8-branes coincide and the theory is in a $SO(32)$ vacuum. The degeneracy of the sub-threshold ground states of a D-particle stuck on the left orientifold plane is either 1 or 32 for the two inequivalent motions of the D8-branes. In one case the D-particle has no strings attached while in the other case (which differs by a single interchange of a D8-brane with its mirror) it is joined by a single χ string to any of the 32 coincident D8-branes.[8] The coupling constant may now be tuned so that in the second case these 32 D-particle states, together with the 32 states of an anti D-particle, give 64 new massless states. These are precisely the 64 states that are needed to enhance the symmetry from $SO(32) \times U(1) \times U_g(1)$ to $SO(34) \times U_g(1)$. The inequivalent $SO(32)$ theory with only two massless D-particle bound states is $SO(32) \times SU(2) \times U(1)_g$. Other examples can be discussed similarly.

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